

Contents

Preface	VII
1 Introduction	1
1.1 Definitions and Classification	1
1.1.1 Purely Viscous or Inelastic Material	3
1.1.2 Perfectly Elastic Material	3
1.1.3 Viscoelastic Material	3
1.2 Non-Newtonian Phenomena	3
1.2.1 The Weissenberg Effect	4
1.2.2 Entry Flow, Extrudate Swell, Melt Fracture, and Vibrating Jet ..	5
1.2.3 Recoil	9
1.2.4 Open Syphon	9
1.2.5 Antithixotropic Effect	10
1.2.6 Drag Reduction	11
1.2.7 Hole Pressure Error	15
1.2.8 Mixing	16
1.2.9 Bubbles, Spheres, and Coalescence	17
2 Material Functions and Generalized Newtonian Fluids	21
2.1 Material Functions	21
2.1.1 Simple Shear Flow	21
2.1.1.1 Steady-State Simple Shear Flow	24
2.1.2 Sinusoidal Shear Flow	28
2.1.3 Transient Shear Flows	32
2.1.3.1 Stress Growth Experiment	32
2.1.3.2 Stress Relaxation Following Steady-Shear Flow	35

2.1.3.3	Stress Relaxation Following a Sudden Deformation . . .	38
2.1.4	Elongational Flow	38
2.1.4.1	Uniaxial Elongation	38
2.1.4.2	Biaxial Elongation	41
2.2	Generalized Newtonian Models	41
2.2.1	Generalized Newtonian Fluid	42
2.2.2	The Power-Law Model	43
2.2.3	The Ellis Model (Bird, Armstrong, and Hassager, 1987)	43
2.2.4	The Carreau Model (1972)	44
2.2.5	The Cross-Williamson Model (1965)	45
2.2.6	The Four-Parameter Carreau Model (Carreau et al., 1979b)	46
2.2.7	The De Kee Model (1977)	46
2.2.8	The Carreau-Yasuda Model (Yasuda, 1979)	48
2.2.9	The Bingham Model (1922)	48
2.2.10	The Casson Model (1959)	49
2.2.11	The Herschel-Bulkley Model (1926)	49
2.2.12	The De Kee-Turcotte Model (1980)	49
2.2.13	The Papanastasiou Model (1987)	51
2.2.14	The Zhu-Kim-De Kee Model (2005)	51
2.2.15	Viscosity Models for Complex Flow Situations.	51
2.3	Thixotropy, Rheopexy, and Hysteresis	52
2.4	Relations Between Material Functions	58
2.5	Temperature, Pressure, and Molecular Weight Effects	61
2.5.1	Effect of Temperature on Viscosity	61
2.5.2	Effect of Pressure on Viscosity	63
2.5.3	Effect of Molecular Weight on Viscosity	64
2.6	Problems	65
2.6.1	Viscosity Data of a PIB Solution ^a	65
2.6.2	Viscosity Data of a CMC Solution ^a	65
2.6.3	The Ellis Model ^a	66
2.6.4	Viscosity Data for a PS Solution ^b	66
2.6.5	Rheological Behavior of Drilling Muds ^b	67
2.6.6	The Cross-Williamson Model ^b	68
2.6.7	Viscosity-Molecular Weight Relationship ^b	68

3	Rheometry	69
3.1	Capillary Rheometry	69
3.1.1	Rabinowitsch Analysis	72
3.1.2	End Effects or Bagley Correction	76
3.1.2.1	Fluid Elasticity from End Corrections	80
3.1.3	Mooney Correction	81
3.1.4	Intrinsic Viscosity Measurements	82
3.1.4.1	Comments	84
3.2	Coaxial-Cylinder Rheometers	85
3.2.1	Calculation of Viscosity	86
3.2.1.1	Non-Newtonian Viscosity	89
3.2.1.2	Comments	90
3.2.2	End-Effect Corrections	91
3.2.3	Normal Stress Determination	92
3.3	Cone-and-Plate Geometry	94
3.3.1	Viscosity Determination	96
3.3.2	Normal Stress Determination	98
3.3.3	Inertial Effects	101
3.3.3.1	Torque Correction	102
3.3.3.2	Normal Force Corrections	103
3.3.4	Criteria for Transient Experiments	105
3.4	Concentric-Disk Geometry	110
3.4.1	Viscosity Determination	111
3.4.2	Normal Stress Difference Determination	112
3.5	Yield Stress Measurements	114
3.5.1	Yield Stress Measurement Methods	116
3.5.1.1	Vane Technique	119
3.5.1.2	Slotted-Plate Technique	120
3.5.1.3	Yield Stress From SAOS data	124
3.6	Problems	125
3.6.1	Rabinowitsch-Type Analysis ^a	125
3.6.2	Rabinowitsch Analysis for a Yield Stress Fluid ^b	126
3.6.3	Viscosity of a High-Density Polyethylene ^a	126

3.6.4	Cone-and-Plate Flow ^b	127
3.6.5	Parallel-Plate Rheometer ^b	127
3.6.6	Falling-Cylinder Viscometer ^b	128
3.6.7	Weissenberg Effect ^a	128
3.6.8	Normal Stress Measurements ^a	129
3.6.9	Normal Stress Determination via Exit Pressure ^b	129
3.6.10	Maxwell Extruder ^a	130
3.6.11	Yield Stress Determination ^b	130
4	Transport Phenomena in Simple Flows	131
4.1	Criteria for Using Purely Viscous Models	131
4.2	Isothermal Flow in Simple Geometries	132
4.2.1	Flow of a Shear-Thinning Fluid in a Circular Tube	133
4.2.2	Film Thickness for the Flow on an Inclined Plane	135
4.2.3	Flow in a Thin Slit	137
4.2.4	Helical Flow in an Annular Section	138
4.2.5	Flow in a Disk-Shaped Mold	141
	4.2.5.1 Velocity Profile	143
	4.2.5.2 Pressure Profile	144
4.3	Heat Transfer to Non-Newtonian Fluids	146
4.3.1	Convective Heat Transfer in Poiseuille Flow	146
	4.3.1.1 L�ev�eque Analysis	147
	4.3.1.2 Corrections for Temperature Effects on the Viscosity	153
4.3.2	Heat Generation in Poiseuille Flow	154
	4.3.2.1 Equilibrium Regime	155
	4.3.2.2 Transition Regime (Approximate Solution)	156
4.4	Mass Transfer to Non-Newtonian Fluids	158
4.4.1	Mass Transfer to a Power-Law Fluid Flowing on an Inclined Plate	159
4.4.2	Mass Transfer to a Power-Law Fluid in Poiseuille Flow	161
4.5	Boundary Layer Flows	165
4.5.1	Laminar Boundary Layer Flow of Power-Law Fluids Over a Plate	165
4.5.2	Laminar Thermal Boundary Layer Flow Over a Plate	170

4.6	Non-Fickian Diffusion	173
4.6.1	Factors Affecting the Mass Transport Process	174
4.6.1.1	Effect of Temperature	174
4.6.1.2	Effect of Permeant and Polymer Structure	175
4.6.1.3	Effect of Mechanical Deformation	177
4.6.2	Theory and Modeling	178
4.7	Problems	182
4.7.1	Pressure Drop in a Tube ^a	182
4.7.2	Generalized Reynolds Number for Poiseuille Flow ^a	182
4.7.3	Flow Characteristics of a Suspension ^a	183
4.7.4	Generalized Non-Newtonian Poiseuille Flow ^b	184
4.7.5	Tolerance in Machining an Extrusion Die ^b	184
4.7.6	Wire Coating ^b	185
4.7.7	Axial Flow Between Two Concentric Cylinders ^b	186
4.7.8	Generalized Couette Flow ^b	186
4.7.9	Velocity Controller ^b	188
4.7.10	Drainage of a Power-Law Fluid ^b	188
4.7.11	Heat Transfer by Convection in a Slit ^b	189
4.7.12	Heat Transfer to a Falling Film ^b	190
4.7.13	Mass Transfer to a Falling Film ^b	191
4.7.14	Heat and Mass Transfer in Boundary Layers ^b	192
4.7.15	Viscoelastic (Non-Fickian) Diffusion ^b	192
5	Linear Viscoelasticity	193
5.1	Importance and Definitions	193
5.2	Linear Viscoelastic Models	194
5.2.1	Maxwell Model	195
5.2.2	Generalized Maxwell Model	202
5.2.3	Unspecified Forms for the Maxwell Model	205
5.2.4	Jeffreys Model	211
5.2.5	Voigt–Kelvin Model	212
5.2.6	Other Linear Models	214
5.3	Relaxation Spectrum	216

5.4	Time–Temperature Superposition	219
5.5	Problems	223
5.5.1	Rheological Model with Friction ^a	223
5.5.2	Maxwell Model ^a	223
5.5.3	Stress Relaxation for a Maxwell Fluid ^a	223
5.5.4	Complex Viscosity of a Generalized Maxwell Fluid ^b	224
5.5.5	The Jeffreys Model ^b	225
5.5.6	Maxwell and Voigt–Kelvin Elements ^b	225
5.5.7	Storage and Loss Moduli of a Voigt–Kelvin Material ^a	226
5.5.8	Complex Compliance ^b	227
5.5.9	Relaxation Modulus ^b	227
6	Non-Linear Viscoelasticity	229
6.1	Non-Linear Deformations	229
6.1.1	Expressions for the Deformation and Deformation Rate	231
6.1.2	Pure Deformation or Uniaxial Elongation	236
6.1.3	Planar Elongation	239
6.1.4	Expansion or Compression	240
6.1.5	Simple Shear	240
6.1.5.1	Comments	241
6.2	Formulation of Constitutive Equations	244
6.2.1	Material Objectivity and Formulation of Constitutive Equations	244
6.2.2	Maxwell Convected Models	245
6.2.3	Generalized Newtonian models	251
6.3	Differential Constitutive Equations	256
6.3.1	De Witt Model	257
6.3.2	Oldroyd Models	258
6.3.3	White–Metzner Model	259
6.3.4	Marrucci Model	267
6.3.5	Phan-Thien–Tanner Model	270
6.4	Integral Constitutive Equations	272
6.4.1	Lodge Model	273
6.4.2	Carreau Constitutive Equation	278
6.4.2.1	Carreau A	280

6.4.2.2	Carreau B	282
6.4.2.3	De Kee Model	286
6.4.3	K-BKZ Constitutive Equation	287
6.4.3.1	Wagner Model	290
6.4.4	LeRoy–Pierrard Equation	294
6.5	Concluding Remarks	298
6.6	Problems	299
6.6.1	Planar Elongational Flow ^a	299
6.6.2	Elongational Viscosity of a Lower-Convected Maxwell Fluid ^a	300
6.6.3	Biaxial Elongation ^b	300
6.6.4	Admissible Constitutive Equations ^a	300
6.6.5	Second-Order Fluid ^b	301
6.6.6	Elongational Viscosity of an Oldroyd-B Fluid ^b	301
6.6.7	Transient Behavior of a White–Metzner Fluid ^b	301
6.6.8	Flow of a White–Metzner Fluid in a Tube Under an Oscillatory Pressure Gradient ^b	301
6.6.9	Viscometric Functions for a Marrucci Fluid ^a	302
6.6.10	Material Functions for a Carreau Fluid ^b	302
6.6.11	Material Functions for a Maxwell Model Involving Slip ^b	303
6.6.12	Relations Between Material Functions ^b	303
6.6.13	Flow Above an Oscillating Plate ^b	303
7	Constitutive Equations from Molecular Theories	305
7.1	Bead-and-Spring-Type Models	306
7.1.1	Hookean Elastic Dumbbell	306
7.1.1.1	Relation Between the Connector Force and the Stress Tensor	307
7.1.1.2	Distribution Function	309
7.1.1.3	Distribution Function $\psi(\mathbf{R}, t)$	311
7.1.1.4	Force Balance on Dumbbells	311
7.1.2	Finitely Extensible Non-Linear Elastic (FENE) Dumbbell	315
7.1.3	Rouse and Zimm Models	319
7.2	Network Theories	329
7.2.1	General Network Concept	329

7.2.2	Rubber-Like Solids	331
7.2.3	Elastic Liquids	333
7.2.4	Other Developments	335
7.3	Reptation Theories	339
7.3.1	The Tube Model	339
7.3.2	Doi-Edwards Model	342
7.3.3	Pom-Pom Models	346
7.3.4	The Curtiss-Bird Kinetic Theory	347
7.4	Conformation Tensor Rheological Models	351
7.4.1	Basic Description of the Conformation Model	351
7.4.2	FENE-Charged Macromolecules	355
7.4.3	Rod-Like and Worm-Like Macromolecules	361
7.4.4	Generalization of the Conformation Tensor Model	370
7.5	Problems	379
7.5.1	Hookean Dumbbell Model ^b	379
7.5.2	Tanner Equation ^a	379
7.5.3	Complex Viscosity of Rouse Fluid ^b	379
7.5.4	Network Model ^b	379
7.5.5	Conformation Model ^b	380
7.5.6	FENE Conformation Model ^b	380
7.5.7	Rod-Like Macromolecules ^b	380
8	Multiphase Systems	381
8.1	Systems of Industrial Interest	381
8.2	Rheology of Suspensions	383
8.2.1	Viscosity of Dilute Suspensions of Rigid Spheres	384
8.2.2	Rheology of Emulsions	387
8.2.2.1	Oldroyd's Emulsion Model	388
8.2.2.2	Choi and Schowalter's Emulsion Model	390
8.2.2.3	Palierne's Model	391
8.2.3	Linear Viscoelasticity of Polymer Blends	393
8.2.4	Rheology of Concentrated Suspensions of Non-Interactive Particles	399
8.2.4.1	Elasticity of Suspensions of Spheres	402

8.2.5	Rheology of Glass Fiber Suspensions	403
8.2.6	Suspensions of Interacting Particles	409
8.2.7	Concluding Remarks	421
8.3	Flow About a Rigid Particle	421
8.3.1	Flow of a Power-Law Fluid Past a Sphere	421
8.3.2	Other Fluid Models	426
8.3.3	Viscoplastic Fluids	426
8.3.4	Viscoelastic Fluids	427
8.3.5	Wall Effects	428
8.3.6	Non-Spherical Particles	430
8.3.7	Drag-Reducing Fluids	431
8.3.8	Behavior in Confined Flows	432
8.4	Flow Around Fluid Spheres	434
8.4.1	Creeping Flow of a Power-Law Fluid Past a Gas Bubble	434
8.4.2	Experimental Results on Single Bubbles	435
8.5	Creeping Flow of a Power-Law Fluid Around a Newtonian Droplet	438
8.5.1	Experimental Results on Falling Drops	440
8.6	Flow in Packed Beds	440
8.6.1	Creeping Power-Law Flow in Beds of Spherical Particles: The Capillary Model	440
8.6.2	Other Fluid Models	445
8.6.3	Viscoelastic Effects	445
8.6.4	Wall Effects	447
8.6.5	Effects of Particle Shape	448
8.6.6	“Submerged Objects” Approach to Fluid Flow in Packed Beds: Creeping Flow	449
8.7	Fluidized Beds	451
8.7.1	Minimum Fluidization Velocity	451
8.7.2	Bed Expansion Behavior	454
8.7.3	Heat and Mass Transfer in Packed and Fluidized Beds	456
8.8	Problems	457
8.8.1	Einstein’s Result ^b	457
8.8.2	Oldroyd’s Emulsion Model ^b	458

8.8.3	Palierne's Emulsion Model ^b	458
8.8.4	Flow About a Sphere ^b	458
8.8.5	Friction Factor for a Packed Bed ^b	459
8.8.6	Criterion for Flow in a Viscoplastic Fluid ^a	459
9	Liquid Mixing	461
9.1	Introduction	461
9.2	Mechanisms of Mixing	463
9.2.1	Laminar Mixing	463
9.2.2	Turbulent Mixing	466
9.3	Scale-Up and Similarity Criteria	466
9.4	Power Consumption in Agitated Tanks	472
9.4.1	Low-Viscosity Systems	472
9.4.2	High-Viscosity Inelastic Fluids	474
9.4.3	Viscoelastic Systems	491
9.5	Flow Patterns	493
9.5.1	Class I Agitators	493
9.5.2	Class II Agitators	495
9.5.3	Class III Agitators	498
9.6	Mixing and Circulation Times	501
9.7	Gas Dispersion	504
9.7.1	Gas Dispersion Mechanisms	505
9.7.2	Power Consumption in Gas-Dispersed Systems	507
9.7.3	Bubble Size and Holdup	510
9.7.4	Mass Transfer Coefficient	511
9.8	Heat Transfer	512
9.8.1	Class I Agitators	514
9.8.2	Class II Agitators	515
9.8.3	Class III Agitators	517
9.9	Mixing Equipment and its Selection	519
9.9.1	Mechanical Agitation	519
9.9.1.1	Tanks	519
9.9.1.2	Baffles	520

9.9.1.3	Impellers	520
9.9.2	Extruders	522
9.10	Problems	523
9.10.1	Power Requirement for Shear-Thinning Fluids ^a	523
9.10.2	Effective Deformation Rate ^a	524
9.10.3	Bottom Effects on the Metzner–Otto Constant ^a	524
9.10.4	Effective Deformation Rate in the Transition Regime ^b	524
10	Appendix A: General Curvilinear Coordinate Systems and Higher-Order Tensors	525
10.1	Cartesian Vectors and the Summation Convention	525
10.2	General Curvilinear Coordinate Systems	529
10.2.1	Generalized Base Vectors	529
10.2.2	Transformation Rules for Vectors	533
10.2.2.1	Contravariant Transformation	534
10.2.2.2	Covariant Transformation	535
10.2.3	Tensors of Arbitrary Order	536
10.2.4	Metric and Permutation Tensors	539
10.2.5	Physical Components	542
10.3	Covariant Differentiation	546
10.3.1	Definitions	546
10.3.2	Properties of Christoffel Symbols	548
10.3.3	Rules of Covariant Differentiation	549
10.3.4	Grad, Div, and Curl	553
10.4	Integral Transforms	559
10.5	Isotropic Tensors, Objective Tensors, and Tensor-Valued Functions	561
10.5.1	Isotropic Tensors	561
10.5.2	Objective Tensors	563
10.5.3	Tensor-Valued Functions	565
10.6	Problems	569
10.6.1	Rotation of Axes ^a	569
10.6.2	Contraction ^a	569
10.6.3	Quotient Law ^a	569

10.6.4	Transformation Rule for the Contravariant Components of a Second-Order Tensor ^a	570
10.6.5	Christoffel Symbols ^a	570
10.6.6	Cylindrical Coordinates ^a	570
10.6.7	Covariant Derivative ^a	570
10.6.8	Physical Components ^a	571
10.6.9	Divergence Theorem ^b	571
10.6.10	Isotropic Tensor ^b	571
10.6.11	Objectivity ^b	571
10.6.12	Invariants ^a	572
10.6.13	Tensor-Valued Function ^b	572
10.6.14	Elongational Flow ^b	572
11	Appendix B: Equations of Change	573
11.1	The Equation of Continuity in Three Coordinate Systems	573
11.2	The Equation of Motion in Rectangular Coordinates (x, y, z)	573
11.2.1	In Terms of σ	573
11.2.2	In Terms of Velocity Gradients for a Newtonian Fluid with Constant ρ and μ	574
11.3	The Equation of Motion in Cylindrical Coordinates (r, θ, z)	574
11.3.1	In Terms of σ	574
11.3.2	In Terms of Velocity Gradients for a Newtonian Fluid with Constant ρ and μ	575
11.4	The Equation of Motion in Spherical Coordinates (r, θ, ϕ)	576
11.4.1	In Terms of σ	576
11.4.2	In Terms of Velocity Gradients for a Newtonian Fluid with Constant ρ and μ	576
	References	579
	Notation	599
	Index	611